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PORTO

WORKING PAPERS

ECONOMICS

Nº 05/2019

**OVERLAPPING OWNERSHIP, ENDOGENOUS QUALITY,
AND WELFARE**

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November 2019

Abstract

This paper investigates how overlapping ownership affects quality levels, consumer surplus, firms' profits and welfare when the industry is a vertically differentiated duopoly and quality choice is endogenous. This issue is particularly relevant since recent empirical evidence suggests that overlapping ownership constitutes an important feature of a multitude of vertically differentiated industries. We show that overlapping ownership while detrimental for welfare, may increase or decrease the quality gap, consumer surplus and firms' profits. In particular, when the overlapping ownership structure is such that the high quality firm places a positive weight on the low quality firm's profits, the incentives of the high quality firm to compete aggressively reduce. This may increase the equilibrium quality of the low quality firm, which in turn may lead to higher consumer surplus, despite higher prices.

JEL Classification: L13; L41.

Keywords: Overlapping Ownership, Vertical Differentiation.

*Duarte Brito (dmb@fct.unl.pt) gratefully acknowledges financial support from Fundação para a Ciência e a Tecnologia (UID/ECO/04007/2019). Ricardo Ribeiro (rribeiro@porto.ucp.pt) gratefully acknowledges financial support from Fundação para a Ciência e a Tecnologia (UID/GES/00731/2019). Helder Vasconcelos (hvasconcelos@fep.up.pt) gratefully acknowledges financial support from Fundação para a Ciência e a Tecnologia (UID/ECO/04105/2019). All remaining errors are of course our own.

1 Introduction

Overlapping ownership - in the form of cross-ownership by competitors (internal shareholders) or common ownership by (external) shareholders - can induce managers to internalize the externalities they impose on rival firms (Rubinstein and Yaari, 1983; Rotemberg, 1984; Gordon, 1990; Hansen and Lott, 1996). This internalization can naturally lessen product market competition since it reduces the incentive of firms with ownership links to compete aggressively, leading (i) to higher product prices and lower output levels (Bresnahan and Salop, 1986; Reynolds and Snapp, 1986; Flath, 1992; Dietzenbacher, Smid and Volkerink, 2000; Shelegia and Spiegel, 2012; Brito, Ribeiro and Vasconcelos, 2019a);¹ and (ii) to a lower likelihood of entry (Newham, Selde-slachts and Banal-Estanol, 2018). However, this internalization can also have a bright side by (i) promoting cost-reducing investments (Shelegia and Spiegel, 2015; Antón *et al.*, 2018; López and Vives, 2019); (ii) facilitating the transfer of tacit knowledge and product innovation (Ghosh and Morita, 2017; Papadopoulos, Petrakis and Skartados, 2019); and (iii) reducing intra-industry portfolio risks (Shy and Stenbacka, 2019).

We contribute to this strand of the literature by studying the effects of overlapping ownership on the quality choices, consumer surplus, profits and welfare of a vertically differentiated duopoly. This issue is particularly relevant since recent empirical evidence suggests that overlapping ownership constitutes an important feature of a multitude of vertically differentiated industries. See, for example, Schmalz (2018), Newham, Selde-slachts and Banal-Estanol (2018) and Backus, Conlon and Sinkinson (2019) for evidence on the airline, banking, supermarket and pharmaceutical industries.^{2,3} We show that when the overlapping ownership structure is such that the high quality firm places a positive weight on the low quality firm's profits, the incentives of the high quality firm to compete aggressively reduce. This may increase the equilibrium quality of the low quality firm, which in turn may lead to higher consumer surplus, despite higher prices.

The model, the equilibria and the conclusions are presented in sections 2, 3 and 4, respectively.

¹ Brito, Ribeiro and Vasconcelos (2019a) show that overlapping ownership can induce product prices and output levels that are even higher and lower, respectively, than those in a monopoly.

² For a characterization of the importance of overlapping ownership in those industries please see Tables 2, 3 and 4 in Schmalz (2018), Table 1 in Newham, Selde-slachts and Banal-Estanol (2018) and Figure 12 in Backus, Conlon and Sinkinson (2019).

³ We can identify features of an airline such as bag handling, gate location, connecting layover times, flight schedules, in-flight services, legroom, seat characteristics, and flight frequency with the quality of an airline (Barbot, 2004; Brueckner and Flores-Fillol, 2019). We can identify the probability of failure of a bank with the quality of the bank (Vives, 2016). We can identify features of a supermarket such as product assortment, store location, product availability, car parking space, and opening hours with the quality of a supermarket (Aslan, 2019). We can identify the brand name of a pharmaceutical product (even though generics are legislated to be therapeutically identical to branded products) with the perceived quality of the product (Cabral, 2003).

All proofs are presented in the mathematical appendix.

2 Theoretical Model

We follow Wauthy (1996)'s approach and notation. Two duopolists, firm 1 and firm 2, sell products of different quality to a continuum of consumers that have different valuations for quality. We assume that each consumer is identified by a parameter θ that characterizes the utility when purchasing from firm $i = L, H$, as follows: $u_{\theta i} = \theta s_i - p_i$, where s_i and p_i denote the quality and price of firm i . θ is uniformly distributed over the support $[\theta^-, \theta^+]$, and θ^+/θ^- is assumed to be sufficiently large so that the market is not covered in equilibrium. We focus on the non-trivial case in which $s_H > s_L$, with s_H and s_L denoting the quality level of the high (H) and low (L) quality firm, respectively. The utility of not purchasing any product (outside option $i = 0$) is normalized to zero: $u_{\theta 0} = 0$.⁴

We assume that quality is costless and can take values in interval $[0, s^+]$ in the lines of Choi and Shin (1992) and Wauthy (1996). This simplifies the analysis considerably. Assuming fixed or variable costs of quality as in Motta (1993) makes the model intractable and it is no longer possible to solve explicitly for the equilibrium quality levels. This constitutes a very interesting potential area for future research.

We also assume that, due to overlapping ownership, firm i 's objective function places a weight $w_i < 1$ on firm j 's profit (with the weight on own profit normalized to 1). These assumptions imply that the objective function of firm $i = L, H$ is $\hat{\pi}_i = \pi_i + w_i \pi_j = p_i D_i + w_i p_j D_j$, where π_i and D_i denote the profit and demand of firm i .⁵

3 Game, Timing and Equilibrium

Consumers and firms play the following game. At the beginning, nature draws the valuations of each consumer for quality. Next, firms address a two-stage decision problem. In the first (second) stage, each firm chooses the quality (price) of its product. Finally, each consumer selects the option ($i = H, L, 0$) that provides the highest utility. We focus on the sub-game perfect Nash equilibrium (SPNE) of the game and begin by addressing the consumers decision problem.

⁴If, alternatively, the market was fully covered, the outcome would be maximum differentiation, regardless of the ownership structure and prices would increase in the presence of overlapping ownership.

⁵We are agnostic about whether overlapping ownership is induced by common-ownership, cross-ownership or both and about which particular type of weight is used. See Brito *et al.* (2018) for a review of the implications of each type of ownership on the objective function of firms. See Backus, Conlon and Sinkinson (2019) and Brito *et al.* (2019b) for a discussion of different alternative weights.

3.1 Consumers Decision Problem

It is straightforward to show that consumers with (i) $\theta \geq \theta_{HL} = \frac{p_H - p_L}{s_H - s_L}$ will purchase the high quality product; (ii) $\frac{p_L}{s_L} = \theta_{L0} \leq \theta < \theta_{HL} = \frac{p_H - p_L}{s_H - s_L}$ will purchase the low quality product; and (iii) $\theta < \theta_{L0} = \frac{p_L}{s_L}$ will choose not to purchase. This yields the following demand functions:

$$\begin{aligned} D_H &= \frac{1}{\theta^+ - \theta^-} \left(\theta^+ - \frac{p_H - p_L}{s_H - s_L} \right) \\ D_L &= \frac{1}{\theta^+ - \theta^-} \left(\frac{p_H - p_L}{s_H - s_L} - \frac{p_L}{s_L} \right). \end{aligned} \quad (1)$$

3.2 Firms Decision Problem

The SPNE of the game involving the firms' decision problem is obtained by backward induction. In the second stage, the two firms (simultaneously) set the prices that maximize their objective function given the quality levels. Lemma 1 presents the corresponding equilibrium.

Lemma 1 *The equilibrium prices, as a function of the quality levels, are:*

$$\begin{aligned} p_H &= \frac{2(s_H - s_L)s_H}{4s_H - s_L(w_L + 1)(w_H + 1)} \theta^+ \\ p_L &= \frac{(w_L + 1)(s_H - s_L)s_L}{4s_H - s_L(w_L + 1)(w_H + 1)} \theta^+. \end{aligned}$$

Given the quality levels s_H and s_L , both equilibrium prices increase in w_L and w_H . The fact that each firm places a positive weight on the rival's profit makes them price less aggressively. The price difference $p_H - p_L$ increases with w_H and decreases with w_L because own equilibrium price is more affected by an increase in the weight given to the rival than the rival's price.

Having addressed the equilibrium in the pricing stage, we now address the quality stage. The two firms (simultaneously) set quality levels anticipating the price equilibrium above. Lemma 2 presents the corresponding equilibrium.⁶

Lemma 2 *In equilibrium, firms set the following quality levels:*

$$\begin{aligned} s_H &= s^+ \\ s_L &= \frac{4(1 - w_L)^2}{(w_L + 1)(w_H^2 2w_L(w_L + 1) - w_H(10w_L - 3w_L^2 + 1) - 4w_L + w_L^2 + 7)} s^+. \end{aligned}$$

Corollary 1 *For any $(w_L, w_H) \in (0, 1)^2$, s_H is invariant to w_H and w_L , while s_L is increasing in w_H and decreasing in w_L .*

⁶The condition for an uncovered market is $\frac{\theta^+}{\theta^-} > \frac{(1 - w_L w_H)8}{1 - w_L(2w_H + w_L + 2w_H w_L - 4)}$.

In order to discuss the implications of Corollary 1 on consumer surplus, firms' profits and welfare, we begin (for tractability) by analyzing some particular cases before addressing the general case.

3.2.1 Benchmark Case: $w_L = w_H = 0$

In the absence of overlapping ownership, Lemmas 1 and 2 imply that:

$$\begin{aligned} s_H &= s^+ & s_L &= \frac{4}{7}s^+ \\ p_H &= \frac{1}{4}s^+\theta^+ & p_L &= \frac{1}{14}s^+\theta^+. \end{aligned} \tag{2}$$

This yields, as in Choi and Shin (1992) and Wauthy (1996), that the lower quality firm chooses quality and price levels which are $4/7$ and $2/7$, respectively, of those of the higher quality firm. As a consequence, $\theta_{HL} = \frac{5}{12}\theta^+$ and $\theta_{L0} = \frac{1}{8}\theta^+$, which yields that $D_H = \frac{7\theta^+}{12(\theta^+ - \theta^-)}$ and $D_L = \frac{7\theta^+}{24(\theta^+ - \theta^-)}$. In turn, consumer surplus is $CS = \frac{7s^+\theta^+}{24(\theta^+ - \theta^-)}$ and, since costs are zero, firms' profits are $\pi_H = \frac{7s^+\theta^+}{48(\theta^+ - \theta^-)}$ and $\pi_L = \frac{s^+\theta^+}{48(\theta^+ - \theta^-)}$, which implies that welfare is $W = \frac{11s^+\theta^+}{24(\theta^+ - \theta^-)}$.

3.2.2 Case 1: $w_H > 0$ and $w_L = 0$

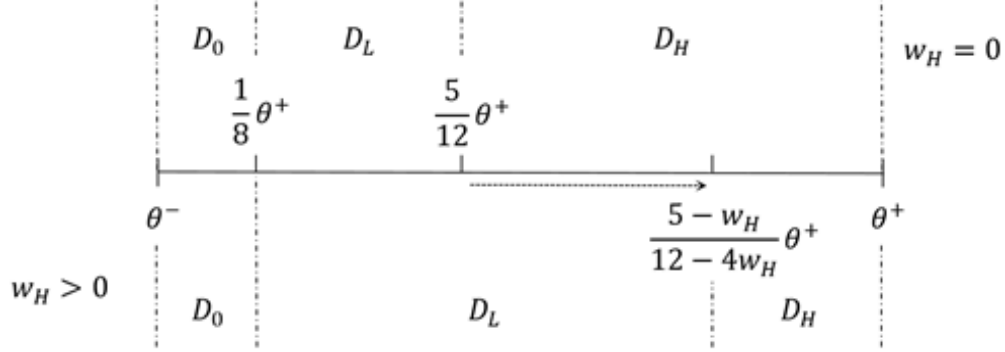
In this case, the overlapping ownership structure is such that only the high quality firm places a positive weight on the low quality firm's profits. Lemmas 1 and 2 imply that:

$$\begin{aligned} s_H &= s^+ & s_L &= \frac{4}{7-w_H}s^+ \\ p_H &= \frac{1}{4}s^+\theta^+ & p_L &= \frac{1}{14-2w_H}s^+\theta^+. \end{aligned} \tag{3}$$

This yields, as established by Corollary 1, that the high quality firm chooses the same equilibrium quality (and price) as in the benchmark case while the low quality firm has an incentive to increase its equilibrium quality (and price) and narrow the quality gap. The reason being that the high quality firm will now price less aggressively (given it internalizes the externality imposed on the rival), which makes the demand of the low quality firm more sensitive to its quality level.⁷ As a consequence, θ_{HL} increases while θ_{L0} remains unchanged: $\theta_{HL} = \frac{5-w_H}{12-4w_H}\theta^+$ and $\theta_{L0} = \frac{1}{8}\theta^+$, which

⁷In the benchmark case, the choice of quality level by the low quality firm is the result of two countervailing effects. The first (direct) effect induces the firm to increase its quality level since higher quality increases demand. The second (strategic effect) induces the firm to decrease its quality level since higher quality leads the rival (the high quality firm) to lower its price, which in turn decreases demand for the firm. In case 1, overlapping ownership increases the first effect (since it makes the demand of the low quality firm more sensitive to its quality level, as the high quality firm will price less aggressively) and may increase or decrease the second effect (since it can make the price of the high quality firm more or less sensitive to the quality level of the low quality firm, depending on the quality levels). The impact on the first effect dominates and so the quality level of the low quality firm, increases.

FIGURE 1
Demand Impacts in Case 1



yields that some demand will now be diverted from the high quality firm to the low quality firm, as depicted in Figure 1. Proposition 1 discusses the impact on consumer surplus, firms profits and welfare.

Proposition 1 *If $w_L = 0$:*

- (a) *Consumer surplus increases with w_H ;*
- (b) *The high (low) quality firm's profit decreases (increases) with w_H ;*
- (c) *Welfare decreases with w_H .*

We begin by addressing the impact on consumer surplus. It is striking it may increase when there is overlapping ownership and competition is less intense. The reason is as follows. When comparing the equilibrium decisions when $w_H = 0$ with those when $w_H > 0$ one can divide consumers into four distinct groups, as depicted in Figure 1. Consumers with $\theta < \frac{1}{8}\theta^+$ and $\theta \geq \frac{5 - w_H}{12 - 4w_H}\theta^+$ are unaffected by the change in w_H . The former do not make any surplus while the latter make the same surplus because s_H and p_H do not change with w_H . Consumers with $\frac{1}{8}\theta^+ \leq \theta < \frac{5}{12}\theta^+$ and $\frac{5}{12}\theta^+ \leq \theta < \frac{5 - w_H}{12 - 4w_H}\theta^+$, on the other hand, benefit from the change in w_H . The former now pay a higher price, but are more than compensated by the higher quality, while the latter now purchase a lower quality, but are more than compensated by the lower price.

We now address the impact on firms' profits. The profit of the high quality firm decreases with w_H because some demand is diverted from the high quality firm to the low quality firm while p_H does not change. The high quality firm accepts this loss in profit since it now places a positive weight on the profit of the low quality firm, which increases with w_H because some demand is diverted from the high quality firm to the low quality firm while p_L increases.

Finally, we address the impact on welfare. Despite the positive impact on consumers, welfare

decreases with w_H as the result of the following trade-off (since price is irrelevant in terms of welfare). On the one hand, some consumers continue to buy the low quality product, whose quality increases with w_H . On the other hand, some consumers switch from the high quality product to the low quality one. The second effect, which is felt by consumers with a higher valuation for quality, dominates.

3.2.3 Case 2: $w_H = 0$ and $w_L > 0$

In this case, the overlapping ownership structure is such that only the low quality firm places a positive weight on the high quality firm's profits. Lemmas 1 and 2 imply that:

$$\begin{aligned} s_H &= s^+ & s_L &= \frac{4(w_L-1)^2}{(w_L+1)(7-4w_L+w_L^2)} s^+ \\ p_H &= \frac{1+4w_L-w_L^2}{4+4w_L} s^+ \theta^+ & p_L &= \frac{(1-w_L)^2(1+4w_L-w_L^2)}{2(w_L+1)(7-4w_L+w_L^2)} s^+ \theta^+. \end{aligned} \quad (4)$$

This yields, as established by Corollary 1, that the high quality firm chooses the same equilibrium quality (and a higher price) while the low quality firm has an incentive to decrease its equilibrium quality (and increase price for $w_L < 0.25605$ /decrease price for $w_L > 0.25605$), widening the quality gap. The reason being that such lower quality level benefits the high quality firm, which is now internalized by the low quality firm.⁸ As a consequence, θ_{HL} and θ_{L0} increase: $\theta_{HL} = \frac{5-w_L^2}{12-4w_L} \theta^+$ and $\theta_{L0} = \frac{1+4w_L-w_L^2}{8} \theta^+$, which yields that the demand decreases for the two firms, as depicted in Figure 2. Panel A depicts the case (labeled 2a) in which the new (increased) θ_{L0} is lower than $\frac{5}{12} \theta^+$ (i.e., when $0 < w_L < 0.70901$) while Panel B depicts the case (labeled 2b) in which the new (increased) θ_{L0} is greater than $\frac{5}{12} \theta^+$ (i.e., when $w_L > 0.70901 > 0$). Proposition 2 discusses the impact on consumer surplus, firms profits and welfare.

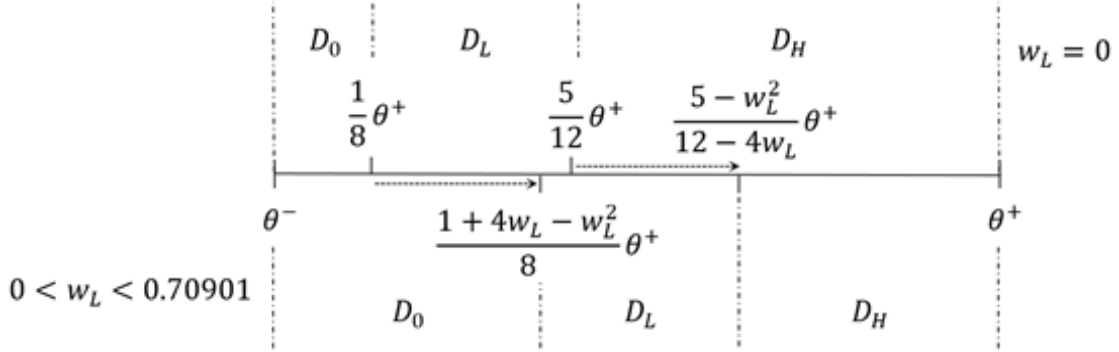
Proposition 2 *If $w_H = 0$:*

- (a) *Consumer surplus decreases with w_L ;*
- (b) *The high (low) quality firm's profit increases (can either increase or decrease) with w_L ;*
- (c) *Welfare decreases with w_L .*

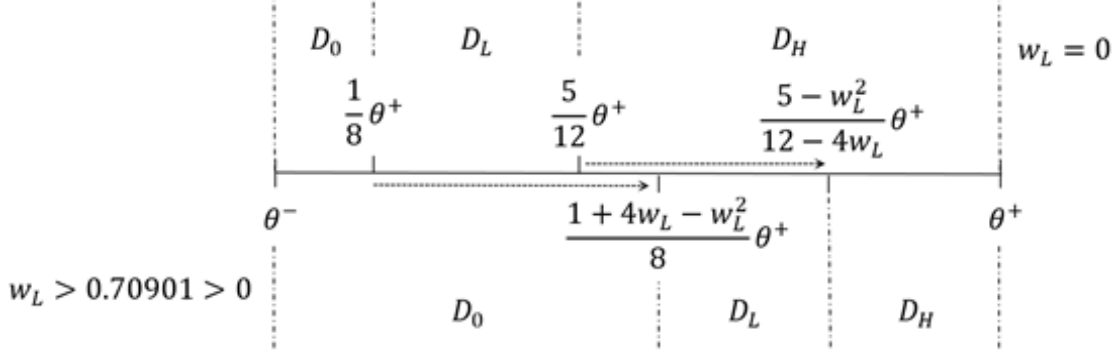
We begin by addressing the impact on consumer surplus. Now, when comparing the equilibrium decisions when $w_L = 0$ with those when $w_L > 0$ one can divide consumers into five distinct groups,

⁸In case 2, overlapping ownership may increase or decrease the two effects discussed in footnote 7 (since it can make the demand of the low quality firm and the price of the high quality firm more or less sensitive to the quality level of the low quality firm, depending on the quality levels). Moreover, it introduces a third (direct) effect, which - dominates and - induces the low quality firm to decrease its quality level, since a lower quality increases the demand for the high quality firm, now internalized by the low quality firm. The high quality firm responds to this increased demand by increasing its price.

FIGURE 2
Panel A: Demand Impacts in Case 2a



Panel B: Demand Impacts in Case 2b



as depicted in Figure 2. Consumers with $\theta < \frac{1}{8}\theta^+$ are unaffected by the change in w_L since they do not make any surplus. All the remaining consumers are, on the other hand, hurt by the change in w_L . Consumers with $\frac{1}{8}\theta^+ \leq \theta < \frac{1+4w_L-w_L^2}{8}\theta^+$ (in case 2a or with $\frac{1}{8}\theta^+ \leq \theta < \frac{5}{12}\theta^+$ and $\frac{5}{12}\theta^+ \leq \theta < \frac{1+4w_L-w_L^2}{8}\theta^+$ in case 2b) are hurt because they make a surplus purchasing from the low quality firm when $w_L = 0$ while they do not make a surplus when $w_L > 0$. Consumers with $\frac{1+4w_L-w_L^2}{8}\theta^+ \leq \theta < \frac{5}{12}\theta^+$ and $\frac{5}{12}\theta^+ \leq \theta < \frac{5-w_L^2}{12-4w_L}\theta^+$ (in case 2a or with $\frac{1+4w_L-w_L^2}{8}\theta^+ \leq \theta < \frac{5-w_L^2}{12-4w_L}\theta^+$ in case 2b) are hurt because now purchase a lower quality but are not compensated price-wise (even when they pay a lower price). Finally, consumers with $\theta \geq \frac{5-w_L^2}{12-4w_L}\theta^+$ are hurt because now purchase the same quality at a higher price.

We now address the impact on firms' profits. The profit of the high quality firm increases with w_L since the decrease in s_L allows the firm to increase p_H , which more than compensates the resulting decrease in demand. The profit of the low quality firm may increase or decrease with w_L because demand decreases (the diversion to the outside option more than compensates the diversion

from the high quality firm) while p_L may increase or decrease.

Finally, we address the impact on welfare. Welfare decreases with w_L because (since price is irrelevant in terms of welfare) some consumers switch from the low quality product to the outside option, some consumers continue to buy the low quality product, whose quality decreases, and some consumers switch from the high quality product to the low quality one.

3.2.4 Case 3: $w_H = w_L = w$

In this case, both firms place a positive and symmetric weight on the rival profit, which combines the two previous cases. Lemmas 1 and 2 imply that:

$$\begin{aligned} s_H &= s^+ & s_L &= \frac{4}{(w+1)^2(2w+7)} s^+ \\ p_H &= \frac{5w+2w^2+1}{4(w+1)^2} s^+ \theta^+ & p_L &= \frac{5w+2w^2+1}{2(2w+7)(w+1)^3} s^+ \theta^+. \end{aligned} \tag{5}$$

This yields that the high quality firm chooses the same equilibrium quality (and a higher price) while the low quality firm has an incentive to decrease its equilibrium quality (and increase price for $w < 0.56986$ /decrease price for $w > 0.56986$). This, in turn, suggests that the dominating effect when both firms places a positive and symmetric weight on the rival profit is the one resulting from w_L , which is, in fact, established in Proposition 3.

Proposition 3 *If $w_H = w_L = w$:*

- (a) *Consumer surplus decreases with w ;*
- (b) *The high (low) quality firm's profit increases (can either increase or decrease) with w ;*
- (c) *Welfare decreases with w .*

3.2.5 General Case

The cases above illustrate that overlapping ownership (a) may increase or decrease consumer surplus (in particular, it decreases consumer surplus if w_L is significantly different from zero);⁹ (b) may increase or decrease firms' profits; and (c) decreases welfare.

⁹The online mathematical appendix shows that a sufficient condition for consumer surplus to decrease is $w_L > 0.013536$.

4 Conclusions

In this paper, we have analyzed the implications of overlapping ownership in a standard vertical differentiation duopoly model. We have shown that overlapping ownership while detrimental for welfare, may increase or decrease the quality gap, consumer surplus and firms' profits. In particular, when overlapping ownership leads the manager of the high quality firm to place some weight on the low quality firm's profits, the low quality level increases and consumers will benefit from this. The reason being that when the rival prices less aggressively, quality differentiation is not as relevant and the low quality firm narrows the quality gap.

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Mathematical Appendix

In this mathematical appendix, we present the proofs of the results presented in the main text.

Proof of Lemma 1

The first-order conditions for maximization of each firm’s objective function are:

$$\begin{aligned} \frac{\partial \left(p_H \left(\theta^+ - \frac{p_H - p_L}{s_H - s_L} \right) + w_H p_L \left(\frac{p_H - p_L}{s_H - s_L} - \frac{p_L}{s_L} \right) \right)}{\partial p_H} &= \frac{-2p_H + (w_H + 1)p_L + \theta^+(s_H - s_L)}{s_H - s_L} = 0 \\ \frac{\partial \left(p_L \left(\frac{p_H - p_L}{s_H - s_L} - \frac{p_L}{s_L} \right) + w_L p_H \left(\theta^+ - \frac{p_H - p_L}{s_H - s_L} \right) \right)}{\partial p_L} &= \frac{-2s_H p_L + p_H(w_L + 1)s_L}{s_L(s_H - s_L)} = 0, \end{aligned}$$

from where the result follows directly.

Proof of Lemma 2

Plugging the equilibrium prices into the objective functions of the two firms results in:

$$\begin{aligned} \widehat{\pi}_H(s_H, s_L) &= \frac{1}{\theta^+ - \theta^-} \left(s_H (s_H - s_L) \frac{4s_H - s_L w_H (w_L + 1)^2}{(4s_H - s_L (w_L + 1)(w_H + 1))^2} \right) \theta^{+2} \\ \widehat{\pi}_L(s_H, s_L) &= \frac{1}{\theta^+ - \theta^-} \left(s_H (s_H - s_L) \frac{4s_H w_L - s_L (w_L + 1)(w_L + 2w_H w_L - 1)}{(4s_H - s_L (w_L + 1)(w_H + 1))^2} \right) \theta^{+2}. \end{aligned}$$

The first order conditions for firm L are:

$$\frac{\partial \widehat{\pi}_L(s_H, s_L)}{\partial s_L} = s_H^2 \frac{4s_H (w_L - 1)^2 + s_L (w_L + 1) (-2w_H^2 w_L (w_L + 1) + w_H (10w_L - 3w_L^2 + 1) - w_L^2 + 4w_L - 7)}{(\theta^+ - \theta^-) (4s_H - s_L (w_L + 1)(w_H + 1))^3} = 0,$$

from where we obtain the low quality firm’s best response function:

$$s_L^*(s_H) = \frac{4(1 - w_L)^2}{(w_L + 1)(2w_H^2 w_L (w_L + 1) - w_H (10w_L - 3w_L^2 + 1) + w_L^2 - 4w_L + 7)} s_H.$$

We now show that the second-order condition for an interior maximum for firm L always holds:

$$\frac{\partial^2 (\widehat{\pi}_L(s_H, s_L))}{\partial s_L^2} = -\frac{1}{\theta^+ - \theta^-} \frac{2s_H^2 (w_L + 1)^2 f(s_H, s_L)}{(4s_H - s_L (w_L + 1)(w_H + 1))^4},$$

with:

$$f(s_H, s_L) = s_H 4(1 - w_H)(2 - w_L - w_H w_L) + s_L (w_H + 1) (w_H^2 (2w_L^2 + 2w_L) - w_H (-3w_L^2 + 10w_L + 1) + w_L^2 - 4w_L + 7).$$

Note that $f(s_H, s_L) > f(s_L, s_L) = s_L (3 - w_L - w_H (w_L + 1)) (-2w_H^2 w_L - w_H (3w_L - 1) + 5 - w_L)$. The first two terms are positive and the minimum of the last term is 0, obtained when $w_L = w_H = 1$. Thus, $f(s_H, s_L) > 0$ and the second-order conditions always hold.

With respect to firm H , the derivative is:

$$\frac{\partial (\widehat{\pi}_L(s_H, s_L))}{\partial s_H} = \frac{16s_H^3 - 12s_H^2 s_L (w_L + 1)(w_H + 1) + 2s_H s_L^2 (w_L + 1) (w_H (w_L^2 + 3) + w_H^2 (w_L + 1)^2 + 4) - s_L^3 w_H (w_L + 1)^3 (w_H + 1)}{(\theta^+ - \theta^-) (4s_H - s_L (w_L + 1)(w_H + 1))^3}.$$

Evaluated at $s_L^*(s_H)$ this derivative is:

$$\frac{(1 - w_H w_L)(3 - w_H - w_L - w_H w_L)^2 (w_L + 1)^2}{2(4s_H - s_L(w_L + 1)(w_H + 1))^3} \frac{(w_L + 1)^2}{(1 - w_L)^6} g(w_H, w_L) s_L^*(s_H)^3,$$

which is positive if $g(w_H, w_L) > 0$, with:

$$g(w_H, w_L) = -4w_H^3 w_L^2 (w_L + 1)^2 - 4w_H^2 w_L (w_L + 1) (-5w_L + w_L^2 - 2) - w_H (38w_L + 6w_L^2 + 8w_L^3 - 9w_L^4 + 2w_L^5 + 3) - w_L^5 + 6w_L^4 - 14w_L^3 + 12w_L^2 - w_L + 14.$$

In order to see that, in fact, $g(w_H, w_L) > 0$ note that:

$$\frac{\partial g(w_H, w_L)}{\partial w_H} = -12w_H^2 w_L^2 (w_L + 1)^2 + 8w_H w_L (w_L + 1) (2 + 5w_L - w_L^2) - (38w_L + 6w_L^2 + 8w_L^3 - 9w_L^4 + 2w_L^5 + 3)$$

This is an inverted parabola with two roots above 1. So, for any $w_H \in (0, 1)$ it is always negative, meaning that $g(w_H, w_L)$ always decreases with w_H . The minimum is then obtained when $w_H = 1$ and takes value $g(1, w_L) = (2w_L + 3w_L^2 + 11)(1 - w_L)^3 > 0$. As a result, the high quality firm will always want to set the highest admissible quality.

Proof of Corollary 1

The proof follows directly from:

$$\frac{\partial s_L}{\partial w_H} = \frac{4(1 - w_L)^2 (-w_L^2 (4w_H + 3) - w_L (4w_H - 10) + 1)}{(w_L + 1)(w_H^2 2w_L (w_L + 1) - w_H (10w_L - 3w_L^2 + 1) - 4w_L + w_L^2 + 7)^2}.$$

The denominator is always positive and the second term in the numerator is an inverted parabola with a minimum when $w_L = 0$ or $w_L = 1$. In the former case it is equal to 1 and in the latter case it is equal to $8(1 - w_H)$. Therefore, $\frac{\partial s_L}{\partial w_H} > 0$.

As for $\frac{\partial s_L}{\partial w_L}$ we have that:

$$\frac{\partial s_L}{\partial w_L} = \frac{-2(w_L + 1)(1 + 4w_L - w_L^2)w_H^2 + w_H(25w_L - 9w_L^2 + 3w_L^3 + 13) + w_L(-3w_L + w_L^2 + 3) - 17}{\frac{(w_L + 1)^2(-w_H - 4w_L + 3w_H w_L^2 + 2w_H^2 w_L + w_L^2 + 2w_H^2 w_L^2 - 10w_H w_L + 7)^2}{4(1 - w_L)}}.$$

The denominator is always positive and the numerator is an inverted parabola with an unconstrained maximum at $w_H = \frac{25w_L - 9w_L^2 + 3w_L^3 + 13}{4(w_L + 1)(4w_L - w_L^2 + 1)} > 1$. Therefore, the numerator is maximized at $w_H = 1$ and takes value $-6(1 - w_L)^3 < 0$, meaning that $\frac{\partial s_L}{\partial w_L} < 0$.

Proof of Proposition 1

Given the equilibrium price and quality expressions one can easily compute consumer surplus, firms' profits and welfare, which are presented below divided by $\frac{s + \theta + 2}{\theta + -\theta}$:

$$\begin{aligned} CS &= \frac{1}{32} \frac{28 - 9w_H}{3 - w_H} & \pi_H &= \frac{7 - 3w_H}{16(3 - w_H)} \\ \pi_L &= \frac{1}{16(3 - w_H)} & W &= \frac{1}{32} \frac{44 - 15w_H}{3 - w_H}, \end{aligned}$$

with derivatives:

$$\begin{aligned} \frac{\partial CS}{\partial w_H} &= \frac{1}{32(w_H - 3)^2} > 0 & \frac{\partial \pi_H}{\partial w_H} &= -\frac{1}{8(3 - w_H)^2} < 0 \\ \frac{\partial \pi_L}{\partial w_H} &= \frac{1}{16(3 - w_H)^2} > 0 & \frac{\partial W}{\partial w_H} &= -\frac{1}{32(3 - w_H)^2} < 0. \end{aligned}$$

Proof of Proposition 2

Given the equilibrium price and quality expressions one can easily compute consumer surplus, firms' profits and welfare, which

are presented below divided by $s^+\theta^{+2}$:

$$\begin{aligned} CS &= (-4w_L + w_L^2 + 7) \frac{3w_L - 4w_L^2 + w_L^3 - 4}{32(w_L + 1)(w_L - 3)} & \pi_H &= \frac{(7 - 4w_L + w_L^2)(1 + 4w_L - w_L^2)}{16(w_L + 1)(3 - w_L)} \\ \pi_L &= \frac{1}{16} (1 - w_L)^3 \frac{1 + 4w_L - w_L^2}{(w_L + 1)(3 - w_L)} & W &= \frac{13w_L - 20w_L^2 + 18w_L^3 - 8w_L^4 + w_L^5 + 44}{32(w_L + 1)(3 - w_L)}, \end{aligned}$$

with derivatives:

$$\begin{aligned} \frac{\partial CS}{\partial w_L} &= -(1 - w_L) \frac{-153w_L + 30w_L^2 + 38w_L^3 - 21w_L^4 + 3w_L^5 + 167}{32(w_L + 1)^2(3 - w_L)^2} < 0 & \frac{\partial \pi_H}{\partial w_L} &= \frac{1}{8} (1 - w_L)^2 \frac{29 - w_L - 5w_L^2 + w_L^3}{(w_L + 1)^2(w_L - 3)^2} > 0 \\ \frac{\partial \pi_L}{\partial w_L} &= -(1 - w_L)^2 \frac{56w_L + 6w_L^2 - 16w_L^3 + 3w_L^4 - 1}{16(w_L + 1)^2(3 - w_L)^2} > 0 \text{ iff } w_L < 1.7825 \times 10^{-2} & \frac{\partial W}{\partial w_L} &= (1 - w_L) \frac{-81w_L + 54w_L^2 + 30w_L^3 - 21w_L^4 + 3w_L^5 - 49}{32(w_L + 1)^2(w_L - 3)^2} < 0. \end{aligned}$$

Proof of Proposition 3

Given the equilibrium price and quality expressions one can easily compute both firms' profits, consumer surplus CS and welfare W , which are presented below divided by $\frac{s^+\theta^{+2}}{\theta^+ - \theta^-}$:

$$\begin{aligned} CS &= \frac{19w + 18w^2 + 11w^3 + 2w^4 + 14}{16(w + 3)(w + 1)^3} & \pi_H &= \frac{(7w + 2w^2 + 7)(5w + 2w^2 + 1)}{16(w + 3)(w + 1)^3} \\ \pi_L &= \frac{(1 - w)(5w + 2w^2 + 1)}{16(w + 1)^3(w + 3)} & W &= \frac{65w + 66w^2 + 33w^3 + 6w^4 + 22}{16(w + 3)(w + 1)^3}, \end{aligned}$$

with derivatives:

$$\begin{aligned} \frac{\partial CS}{\partial w} &= \frac{-62w + 6w^2 + 10w^3 + w^4 - 83}{16(w + 1)^4(w + 3)^2} < 0 & \frac{\partial \pi_H}{\partial w} &= \frac{13w - 6w^2 - 3w^3 + 28}{8(w + 1)^4(w + 3)^2} > 0 \\ \frac{\partial \pi_L}{\partial w} &= \frac{-23w - 12w^2 + w^3 + w^4 + 1}{8(w + 1)^4(w + 3)^2} > 0 \text{ iff } w < 4.2538 \times 10^{-2} & \frac{\partial W}{\partial w} &= \frac{-82w - 30w^2 + 6w^3 + 3w^4 - 25}{16(w + 1)^4(w + 3)^2} < 0. \end{aligned}$$

Consumer Surplus in the General Case

Consumer surplus CS in the general case is given by:

$$\begin{aligned} CS(w_H, w_L) &= \int_{\frac{p_L}{s_L}}^{\frac{p_H - p_L}{s_H - s_L}} \left(\frac{1}{\theta^+ - \theta^-} \right) (\theta s_L - p_L) d\theta + \int_{\frac{p_H - p_L}{s_H - s_L}}^{\theta^+} \left(\frac{1}{\theta^+ - \theta^-} \right) (\theta s_H - p_H) d\theta \\ &= \frac{4w_H^3 w_L^2 (w_L + 1)^2 - 12w_H^2 w_L (w_L + 1)^2 + w_H (38w_L - 14w_L^2 + 24w_L^3 - 11w_L^4 + 2w_L^5 + 9)}{32(w_L + 1)(w_H + w_L + w_H w_L - 3)(w_H w_L - 1)^2} \frac{s^+\theta^{+2}}{(\theta^+ - \theta^-)} \\ &\quad + \frac{(-4w_L + w_L^2 + 7)(3w_L - 4w_L^2 + w_L^3 - 4)}{32(w_L + 1)(w_H + w_L + w_H w_L - 3)(w_H w_L - 1)^2} \frac{s^+\theta^{+2}}{\theta^+ - \theta^-}. \end{aligned}$$

The derivative of consumer surplus with respect to w_H is given by:

$$\begin{aligned} \frac{\partial CS}{\partial w_H} &= (1 - w_L)^2 \frac{4w_H^3 w_L^2 (w_L + 1)^2 - 2w_H^2 w_L (w_L + 1)(14w_L - 7w_L^2 + 2w_L^3 + 3)}{-32(1 - w_H w_L)^3 (w_L + 1)(w_H + w_L + w_H w_L - 3)^2} \frac{s^+\theta^{+2}}{(\theta^+ - \theta^-)} \\ &\quad + (1 - w_L)^2 \frac{w_H w_L (36w_L - 50w_L^2 + 28w_L^3 - 5w_L^4 + 39) - (56w_L - 80w_L^2 + 54w_L^3 - 17w_L^4 + 2w_L^5 + 1)}{-32(1 - w_H w_L)^3 (w_L + 1)(w_H + w_L + w_H w_L - 3)^2} \frac{s^+\theta^{+2}}{(\theta^+ - \theta^-)}. \end{aligned}$$

The sign of this derivative is given by the inverse of the sign of:

$$\begin{aligned} h(w_H, w_L) &= 4w_H^3 w_L^2 (w_L + 1)^2 - 2w_H^2 w_L (w_L + 1)(14w_L - 7w_L^2 + 2w_L^3 + 3) \\ &\quad + w_H w_L (36w_L - 50w_L^2 + 28w_L^3 - 5w_L^4 + 39) - (56w_L - 80w_L^2 + 54w_L^3 - 17w_L^4 + 2w_L^5 + 1), \end{aligned}$$

and:

$$\frac{\partial h(w_H, w_L)}{\partial w_H} = 12w_H^2 w_L^2 (w_L + 1)^2 - 4w_H w_L (w_L + 1)(14w_L - 7w_L^2 + 2w_L^3 + 3) + w_L (36w_L - 50w_L^2 + 28w_L^3 - 5w_L^4 + 39).$$

This constitutes a U-shaped parabola in w_H with two roots that are larger than 1:

$$\begin{aligned} w_H^+ &= \frac{6w_L(w_L+1) + (1-w_L)(11w_L-2w_L^2+3) + (w_L-1)^2\sqrt{3w_L+4w_L^2+9}}{6w_L(w_L+1)} > 1 \\ w_H^- &= \frac{6w_L(w_L+1) + (1-w_L)(11w_L-2w_L^2+3) - (w_L-1)^2\sqrt{3w_L+4w_L^2+9}}{6w_L(w_L+1)} > 1 \end{aligned}.$$

So, $\frac{\partial h(w_H, w_L)}{\partial w_H} > 0$. As such, for any w_L , $h(w_H, w_L)$ increases with w_H , meaning that for $w_H < 1$, it is smaller than:

$$h(1, w_L) = -(26w_L - 11w_L^2 + 1)(1 - w_L)^3 < 0$$

This implies that $h(w_H, w_L) < 0$ and, as such, that $\frac{\partial CS}{\partial w_H} > 0$. We can now evaluate CS at $w_H = 1$, the highest possible level and compare it with the case of no overlapping ownership:

$$CS(1, w_L) = \frac{1}{64} \frac{(-6w_L + 3w_L^2 + 19)}{w_L + 1} \frac{s^+ \theta^{+2}}{\theta^+ - \theta^-} \leq CS(0, 0) = \frac{7}{24} \frac{s^+ \theta^{+2}}{\theta^+ - \theta^-},$$

which, in turn, implies that $\frac{1}{64} \frac{(-6w_L + 3w_L^2 + 19)}{w_L + 1} < \frac{7}{24}$ or $\frac{37}{9} - \frac{4}{9}\sqrt{85} < w_L < 1$, constitutes a sufficient condition for consumer surplus to decrease with overlapping ownership.